Appendix 5
Deriving uptake equation
(P. de Willigen)

According to De Willigen and Van Noordwijk (1987 - Table 9.1, equ. 12.9) uptake rate is given by:

$$\frac{\rho^2 \Theta \beta}{2\phi\eta} = \frac{(\rho^2 - 1)\bar{c}}{2G(\rho)} \qquad \textbf{[A1]}$$

Now (l.c. page 125):

$$G(\rho) = \frac{1}{2}\left\{\frac{1 - 3\rho^2}{4} + \frac{\rho^4 \ln \rho}{\rho^2 - 1}\right\} \qquad \textbf{[A2]}$$

As normally $\rho \ll 1$

$$G(\rho) \approx \rho^2 \left(-\frac{3}{8} + \frac{1}{2}\ln \rho\right) \qquad \textbf{[A3]}$$

The parameters ρ 1, ϕ 2 and η 3 are given by:

$$\rho = \frac{R_1}{R_0} \qquad 1$$

$$\phi = \frac{D\ \ S_i}{U\ R_0} = \frac{D\theta\beta C_i}{U\ R_0} \qquad 2 \qquad \textbf{[A4]}$$

$$\eta = \frac{H}{R_0} \qquad 3$$

and the dimensionless concentration by:

$$\bar{c} = \frac{C}{C_i} \quad \text{[A5]}$$

where D is the diffusion coefficient (m2.d-1), H is the thickness of the soil layer (m), U is the uptake rate (g.m-2.d-1), R0 the radius of the root (m) and R1 the radius of the soil cylinder surrounding the root. The latter is given by:

$$R_1 = \frac{1}{\sqrt{\pi L_{rv}}} \quad \text{[A6]}$$

The parameter 4 denotes the buffer power of the soil. Substitution of (A2)-(A6) into (A1) leads to:

$$U = \frac{D\bar{C}H}{R_1^2\left(-\frac{3}{8} + \frac{1}{2}\ln\rho\right)} \quad \text{[A7]}$$

The diffusion coefficient is a function of the water content Θ5, according to:

$$D = (a_1\Theta + a_0)\ \Theta\ D_0 \quad \text{[A8]}$$

where D_0 is the diffusion coefficient of the nutrient in question in water, whereas the concentration can be calculated from the amount in the layer Nstock (g.m-2):

$$C = \frac{N_{stock}}{K_a + \Theta} \quad \text{[A9]}$$

Ka being the adsorption constant. Substitution of (A2)-(A9) into (A1) ultimately yields (A10) which is the basis for equation (10) in WaNuLCAS.

$$U = \frac{\pi D_0\ (a_1\Theta + a_0)\ \Theta\ H\ N_{stock}}{(K_a + \Theta)\left[-\frac{3}{8} + \frac{1}{2}\ln\left\{\frac{1}{R_0\ \sqrt{\pi L_{rv}}}\right\}\right]} \quad \text{[A10]}$$