

SpatRain: a simulator of space/time patterns in rainfall for predicting scale dependence of variability of rainfall-related processes

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Abstract

Variations in river flow tend to decrease with increasing area of consideration, partly due to a decrease in temporal correlation of rainfall events across space. Patchiness of rainfall can contribute to an increase of yield stability over space. Existing rainfall simulators tend to focus on station-level time series, not on space/time autocorrelation. The SpatRain simulator described here is constructed to generate time series of rainfall that are fully compatible with existing station-level records of daily rainfall, but yet can represent substantially different degrees of spatial autocorrelation. Calculations start from the assumed spatial characteristics of a single rainstorm pathway, with a trajectory for the core area of the highest intensity and a decrease of rainfall intensity with increasing distance from this core. The simulator can derive daily amounts of rainfall for a grid of observation points by considering the possibility of multiple storm events per day, but not exceeding the long-term maximum of observed station-level rainfall.

Key words: *NetLogo, rainfall, simulator, SpatRain, spatial patterns, temporal patterns.*

1. Introduction

Most studies of rainfall pattern focus either on the time series (degree of autocorrelation) for rainfall at a single point of observation, or on the spatial patterns of average rainfall over a monthly or yearly period. Small-range spatial variability of daily rainfall has been studied extensively in the context of design of rain-gauge networks. While the recommended density of such networks is often much higher than what exists or is feasible, the small-range variability around the existing data tends to be ignored. Yet, if one is interested in the variability of stream and river flow at different scales, the gradual loss of synchrony in runoff-causing rainfall events is probably an important contributor to the increased ‘evenness’ of river flow with increased area of consideration. Hydrological literature reviewed by Rodriguez-Iturbe and Rinaldo (1997) indicates that maximum daily river flow scales with area to the power 0.75, while mean annual discharge scales with area to the power 1. The difference in scaling factor can be explained by a combination of temporary water storage in river valleys and ‘the fact that rains of high intensity rarely cover the entire basin, but are instead widely and irregularly spaced’. While the temporary storage component is under the influence of land use and ‘watershed management’, the second explanation is not. It is thus important to separate the two types of explanation. An explicit representation of space-time patterns of rainfall will allow such analysis.

In situations where details of the rainfall pattern are important for crop growth and yield formation, the gradual decrease in synchrony of rain may confer advantages to households that operate fields across the village domain rather than in a single block. A quantitative approach to

spatial autocorrelation of rainfall is needed to address these issues and a procedure to generate partially correlated time series for neighbouring locations is desirable for simulation studies.

Substantial emphasis has been given to temporal autocorrelation, but if the results are subsequently assumed to hold invariantly across space, or are supposed to be derived from independent multiple domains, the scaling relations of the processes that are related to rainfall may be wrongly assessed. We thus identified a need for a tool that considers variation across both space and time at daily or event scale and developed a spatial rainfall simulator that can be used to overcome these challenges.

2. Description of the model

2.1. Approach to the problem

Design features of SpatRain include:

- the program must specify degrees of spatial correlation of the simulated rainfall based on storm-level properties; and
- the simulated rainfall for any point in the landscape must be consistent with existing data on the frequency distribution of daily rainfall.

Station-level daily records are often the only information available on the distribution of rainfall. Such data can be represented as a series of monthly ‘exceedance’ graphs, derived from the long-term station records. Between months of the year and locations we may expect differences in the intercept with the X-axis or ‘frequency of wet days’ (days with a measurable amount of precipitation), the intercept with the Y-axis or maximum amount of rainfall in a single day recorded in that particular month of the year, and in the curvature of the (monotonously rising) line between these two points.

Conceptually one can imagine a procedure that reshuffles measured daily sequences of rainfall while maintaining the monthly total. It is like rearranging a ‘jackpot’, where the variability of values exposed on the window should follow our expectation. The total set of permutations of 30 sets of ‘jackpot’ with 30 values is already enormous, and among these we can expect to find a substantial variation in degrees of spatial autocorrelation. By generating a sample of these reshuffling results, calculating autocorrelation and then selecting specific configurations, we would meet the key design criteria specified above. The program will, however, be rather cumbersome and time consuming if large areas are to be considered, and the selection of results that meet a specific change in spatial autocorrelation with increasing distance may require a large subset of reshuffling results. More efficient algorithms are desirable, but the ‘jackpot’ analogue shows that the design rules are not mutually incompatible. A more direct approach can be taken if we assign specific spatial properties to single storm events and then adjust the frequency of storms and the intensity of rainfall in the core area of these storms to match the existing station records.

2.2. Spatial consideration

SpatRain starts from the spatial characteristics of a single rainstorm pathway (with a trajectory for the core area of the highest intensity and a decrease of rainfall intensity with increasing distance from this core) and can derive daily amounts of rainfall for a grid of observation points by considering the possibility of multiple storm events per day. Three parameters are used here for describing rainfall in the core area: the length and the width of the core trajectory and the rainfall depth in the core area (I_{\max}). Two further parameters (spread and edge) describe the decrease of rainfall depth (I_d) with increasing distance from the core (d). The combination of these can produce the full scale of ‘homogenous’ to ‘heterogeneous’ types of rain:

$$I_d = I_{\max} * (1 - \exp(-((\text{spread}/d)^{\text{edge}}))) \dots \dots \dots \text{Equation 1}$$

Equation 1 may produce a narrow-spread area of single storm events, on which its wet cells ratio relative to total area (wc) does not match the wet days fraction of that specific month (wd). Hence, we need to allow for multiple storm events, depending on the area fraction wetted by a single event and the time fraction of rainy days at the measurement station level. For spatially independent multiple events on a single day we can derive that probability of dry days on a given month, $1-wd$, should meet the probability of dry cells during single event, $1-wc$, to the power of events number (N):

$$1-wd = (1-wc)^N \dots \dots \dots \text{Equation 2}$$

2.2. Temporal consideration

Patchy rains have less wet fraction than homogeneous rains in space. In order to conserve each cell to having uniform chance of being hit by storms in time, patchy rains should have higher probability to occur than homogeneous rains. Consequently, the probability of storm with N number of events (E) is defined from wet days fraction (wd) by taking wet cells fraction of N storm events (WC) into account:

$$E = wd/WC \dots \dots \dots \text{Equation 3}$$

Storm events will ‘wet’ a number of cells, some at the core intensity and some at a lower intensity. Given a set of parameters for the storm trajectory, we can derive the frequency distribution of rain depth in wetted cells (p), in n classes. Once this is known, the frequency distribution of core intensities (F) can be derived from the observed station level rain intensities (f).

Frequency distributions of f , p and F should have the same class number and interval order. We use the following order to define the class boundary: $[R_{\max} \dots R_{\max} * q^1], [R_{\max} * q^1 \dots R_{\max} * q^2], \dots, [R_{\max} * q^n \dots R_{\min}]$, where R_{\max} is the maximum rainfall, R_{\min} is the minimum rainfall and n is class intervals number. The value of q is ranging from 0 to 1 and calculated as follows:

$$q = \exp(\ln(R_{\min}/R_{\max})/n), 0 \leq q \leq 1 \dots \dots \dots \text{Equation 4}$$

We first need to recognize the combinations of classes p_j and F_k that are compatible with class f_i :

$$f_i \approx \sum(p_j F_k | j, k \sim i), R_{\min} \leq f_i \leq R_{\max}, R_{\min} \leq p_j \leq R_{\max}, R_{\min} \leq F_k \leq R_{\max} \dots\dots\dots \text{Equation 5}$$

For the highest rainfall class there is only one combination, involving the highest class of both p and F that will yield the desired result, but for the other classes there can be several combinations of p and F that yield the same result (the tail end of a big rainfall event, a medium fraction of a medium storm or the core area of a small storm). We can approach it from the top down, but a simpler derivation starts from the observation that for all distributions f, p and F the sum equals 1. By assuming that the resultant (f) comes from the multiplication between p and F, we then get this basic equation:

$$\sum f = \sum p * \sum F \dots\dots\dots \text{Equation 6}$$

From Equation 6, we can derive a criterion for the shape of the p distribution (that depends on assumed storm properties) that is compatible with the targeted f distribution. If at any point sum of f from frequency class 1 to i divided by sum of p from frequency class 1 to i is less than sum of F from frequency class 1 to i-1, F from frequency class i (F_i) would violate the assumption of non-negative subsequent F terms. So, a cross-over of p and f indicates incompatibility of the storm-level assumptions (that generate the p curve) with the station-level rainfall records (that generate the f curve).

3. Application

Here we present application of SpatRain using daily rainfall time series from the meso-scale catchments area of Sumberjaya, Lampung, Sumatra, Indonesia. The area of the catchments is about 500 km². Actual daily records were obtained from 3 rainfall stations located within the catchments. One station has 21-year time series, while other two have shorter time series but are probably not significantly different in means or variance measures (Manik and Sidle, 2003).

Various ‘compatible’ properties of single storm were used, representing storms with high edge, storms with high spread, and storms with large core area. Detail of these properties is listed in Table 1. Spatial patterns produced by the assumed storm properties are shown in Figure 1. From each setting, we simulated daily rainfall over 30x30-km² space for 20 years. Twenty points were selected randomly to analyse temporal and spatial patterns of the simulated rainfall.

[Figure 1 goes about here]

3.1. Temporal patterns

From the simulation results, high edge, high spread and large core storms resulted different dynamics of daily rainfall, but the patterns of the average agree with the observed data (Figure 2). Exceedance probability of daily rainfall resulted by the storms also agree with exceedance probability of the actual records (Figure 3). The results in Table 1 suggest that annual rainfall increases with increasing edge, spread and core area of the storms, but in term of annual target amount, it is still within ‘acceptable’ range of observed station-level annual rainfall in general, *i.e.* 2531±476 mm.

[Figure 2, 3 go about here]

[Table 1 goes about here]

3.2. Spatial patterns

Semivariance analyses (as a function of increasing distance between observation points) were carried out, as a way to characterize the resulting rainfall patterns and identify the storm-level parameters that lead to specified degrees of spatial correlation. Simulated daily rainfall maps from 30 rain days were analysed to construct semivariogram. Twenty points were selected randomly as starting points to calculate their semivariance relative to their neighbouring points at specified distance. We use distance averaged to summarise the semivariograms (Table 1). In general, high edge storms produced spatial patterns with relatively high semivariance. There is no clear correlation between degree of storm edge and semivariance of its resulted patterns. High spread storms produced spatial patterns with relatively low semivariance. There is decrease on semivariance with increasing spread of the storm. Semivariance decreases with increasing area of the storm core.

Another way to characterise the spatial pattern of the simulated rainfall is from the scaling rule of its maximum rainfall. We use power function to identify correlation between area (as the independent variable) and maximum rainfall (as response variable). Here we use the slope of the function (the power) as the scaling indicator. For this purpose, 20 circles with random location and random size were generated to sample the maximum rainfall within the corresponding boundary. Simulated daily rainfall maps from 30 rain days were used for this analysis. The maximum rainfall from the same area size were averaged and confronted against its corresponding area size. From Table 1, high edge storms produced maximum rainfall which scales with area to the power of about 0.04. Consistent with previous results on semivariance, there is also no clear correlation between degree of storm edge and the scaling factor of maximum rainfall. High spread storms produced maximum rainfall which scales with area to the power of about 0.01-0.02. This scale decreases with increasing spread of the storm. Scale of maximum rainfall also decreases with increasing area of the storm core.

4. Conclusion

In term of its ‘reliability’, SpatRain can (within bounds) match a range of assumed spatial patterns of storms with existing station records. In term of its ‘usefulness’, further application of SpatRain with regards to its ability to generate daily spatial distributions of rainfall from its daily temporal distributions is to use simulated daily rainfall data to construct f that can be generated from other simulators based on monthly statistic. SpatRain also offers solution when spatial patterns consideration of rainfall is important for assessment studies (e.g. analysis of river flow and flooding risk and risk analyses on crop growth) in catchments with low density of rain-gauge networks. For catchments with higher density, storm properties should be calibrated with observed data in term of its spatial pattern. This can be done through semivariance or scaling analyses to the observed data.

Some caution is needed in applying the model. The current procedures are only applicable for catchments which rainfall variability is not much affected by elevation (micro- to meso-scale catchments). Thus, extending the model for application in larger area should be carried out with regard to spatial trends in mean rainfall due to effects of elevation.

Model availability

The SpatRain simulator is implemented using NetLogo, a java-based programmable modelling environment for simulating natural and social phenomena (Uri Wilensky, 1999). SpatRain is freely available on NetLogo website (<http://ccl.northwestern.edu/netlogo/>) or on our website (<http://www.worldagroforestrycentre.org/sea/>).

Acknowledgement

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References

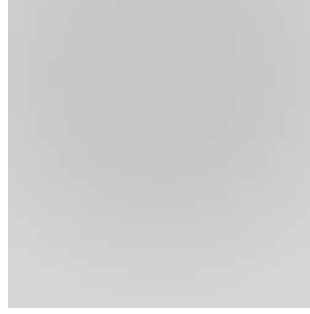
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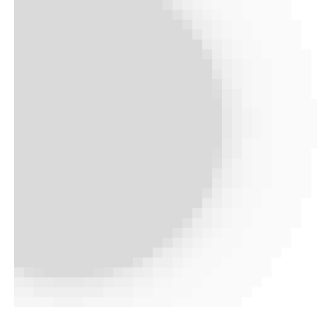
Wilensky, U.. 1999. NetLogo. <http://ccl.northwestern.edu/netlogo/>. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.



(A) High edge

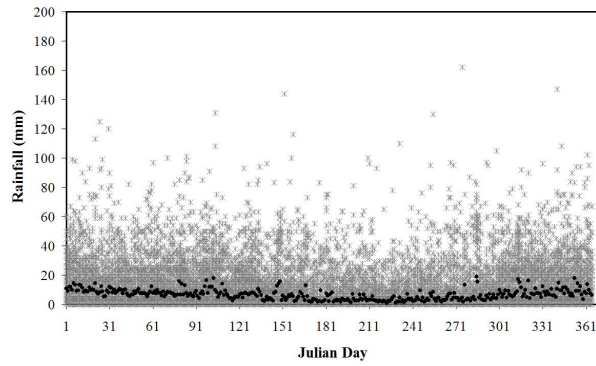


(B) High spread

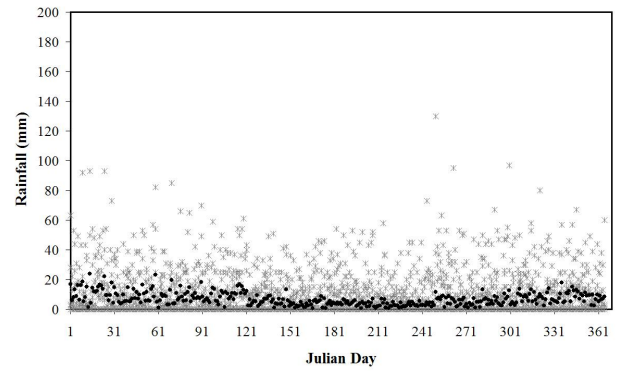


(C) Large core

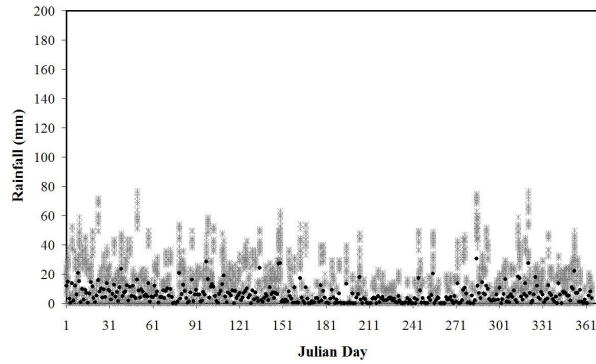
Figure 1. Spatial patterns resulted by: (A) high edge, (B) high spread and (C) large core storms.



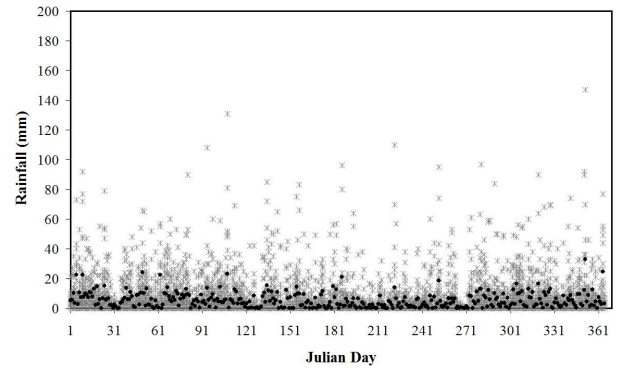
(A) Observed



(B) High edge

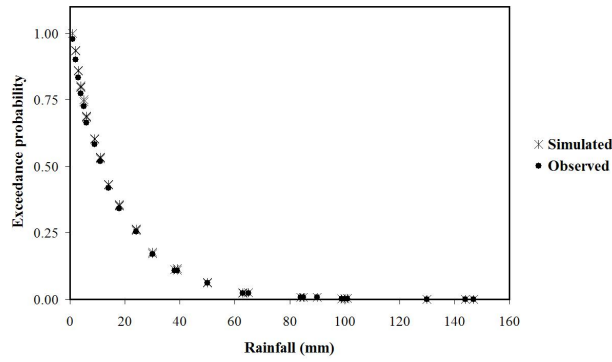


(C) High spread

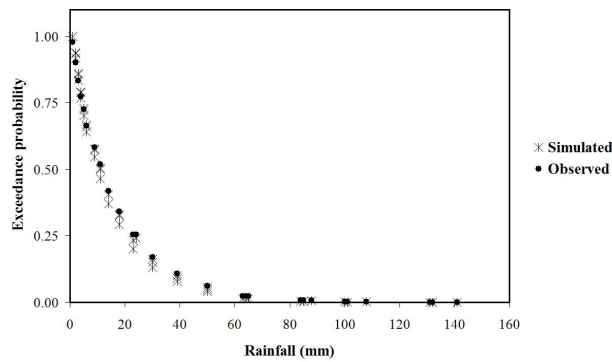


(D) Large core

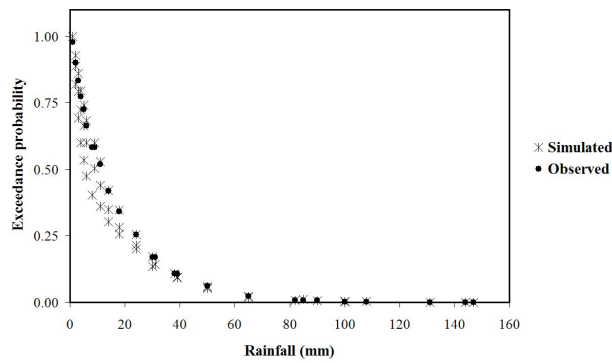
Figure 2. Daily patterns of simulated rainfall (grey crosses) for 20-year period of 20 random points at various storm properties: (B) high edge, (C) high spread and (D) large core storms, compared to actual records (A). Here, black dots are the average of data from the same day.



(A) High edge



(B) High spread



(C) Large core

Figure 3. Exceedance probability of daily rainfall for 20-year period of 20 random points resulted by: (A) high edge, (B) high spread and (C) large core storms, compared to the actual data.

Table 1. Summary of simulated rainfall at various assumed storm properties.

No.	Assumed storm properties				Output summary		
	Spread	Edge	Core width (km)	Core length (km)	Annual rainfall (mm)	Distance averaged semivariance	Maximum rainfall – area relationship
(A) High edge							
1.	1	30	1	1	2522±161	130	$y = 332x^{0.0362}$ $R^2 = 0.76$
2.	1	45	1	1	2540±143	124	$y = 365x^{0.0408}$ $R^2 = 0.73$
3.	1	60	1	1	2546±100	202	$y = 531x^{0.0400}$ $R^2 = 0.67$
(B) High spread							
4.	30	1	1	1	2179±211	3	$y = 333x^{0.0241}$ $R^2 = 0.73$
5.	45	1	1	1	2339±170	1	$y = 347x^{0.0141}$ $R^2 = 0.68$
6.	60	1	1	1	2395±204	0	$y = 452x^{0.0070}$ $R^2 = 0.70$
(C) Large core							
7.	1	1	30	30	1874±178	104	$y = 253x^{0.1179}$ $R^2 = 0.76$
8.	1	1	45	45	2185±243	58	$y = 533x^{0.0287}$ $R^2 = 0.42$
9.	1	1	60	60	2472±160	3	$y = 522x^{0.0038}$ $R^2 = 0.22$